5 (ii) Find the sum of n terms

W.D $\operatorname{cosec}\theta + \operatorname{cosec}2\theta + \operatorname{cosec}2^2\theta + \operatorname{cosec}2^3\theta + \cdots$ to n terms

D.I(U) Maths paper-1st, Group-A summation of series Dr. G. D. Singh

$$t_1 = \operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1+\cos\theta-\cos\theta}{\sin\theta}$$

$$= \frac{2\cos^2\theta/2}{2\sin\theta/2\cdot\cos\theta/2} - \frac{\cos\theta}{\sin\theta}$$

$$\therefore \operatorname{cosec}\theta = \cot\theta/2 - \cot\theta$$

$$\therefore \operatorname{cosec}1\theta = \cot\frac{1}{2}\theta - \cot\theta$$

$$\operatorname{cosec}2\theta = \cot\frac{1}{2}\times 2\theta - \cot 2\theta$$

$$\operatorname{cosec}2^2\theta = \cot\frac{1}{2}\times 2^2\theta - \cot 2^2\theta$$

$$\operatorname{cosec}2^3\theta = \cot\frac{1}{2}\times 2^3\theta - \cot 2^3\theta$$

$$\cdots\quad\cdots\quad\cdots$$

$$\operatorname{cosec}2^{n-1}\theta = \cot\frac{1}{2}\times 2^{n-1}\theta - \cot 2^{n-1}\theta$$

Adding $\operatorname{cosec}\theta + \operatorname{cosec}2\theta + \cdots + \operatorname{cosec}2^{n-1}\theta = \cot\theta/2 - \cot 2^{n-1}\theta$ Ans

$$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$$

$\sin 2A = 2\sin A\cos A$, $\sin A = 2\sin A/2 \cos A/2$

6. w.o. $\dfrac{1}{\cos 1\phi + \cos 3\phi} + \dfrac{1}{\cos\phi + \cos 5\phi} + \dfrac{1}{\cos\phi + \cos 7\phi} + \cdots$ to n terms

$$T_r = \frac{1}{\cos\phi + \cos(2r+1)\phi}$$

(3, 5, 7, ... का rवां पद $= 3 + (r-1)2$
$= 3 + 2r - 2 = (2r+1)$ ✓

or $$T_r = \frac{1}{2\cos\dfrac{(2r+1)\phi + \phi}{2}\cos\dfrac{(2r+1)\phi - \phi}{2}}$$

[$\cos C + \cos D = 2\cos\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

$$= \frac{1}{2\cos\dfrac{(2r+2)}{2}\phi\,\cos\dfrac{2r\phi}{2}}$$

$$T_r = \frac{1}{2\cos(r+1)\phi\cdot\cos r\phi}$$

$$= \frac{1}{2\sin\phi} \times \frac{\sin\phi}{\cos(r+1)\phi\cdot\cos r\phi}$$

$$= \frac{1}{2\sin\phi} \times \frac{\sin\{(r+1)\phi - r\phi\}}{\cos\{(r+1)\phi\}\cos r\phi}$$

$$= \frac{1}{2\sin\phi} \times \frac{\sin(r+1)\phi\cos r\phi - \cos(r+1)\phi\sin r\phi}{\cos(r+1)\phi\cos r\phi}$$

$$= \frac{1}{2\sin\phi}\left[\frac{\sin(r+1)\phi\cos r\phi}{\cos(r+1)\phi\cdot\cos r\phi} - \frac{\cos(r+1)\phi\sin r\phi}{\cos(r+1)\phi\cdot\cos r\phi}\right]$$

$$= \frac{1}{2\sin\phi}\left[\tan(r+1)\phi - \tan r\phi\right]$$

$$\therefore T_r = \frac{1}{2\sin\phi}\left[\tan(r+1)\phi - \tan r\phi\right]$$

$$T_1 = \frac{1}{2\sin\phi}\left[\tan 2\phi - \tan\phi\right]$$

$$T_2 = \frac{1}{2\sin\phi}\left[\tan 3\phi - \tan 2\phi\right]$$

$$T_3 = \frac{1}{2\sin\phi}\left[\tan 4\phi - \tan 3\phi\right]$$

$$\cdots\cdots$$

$$T_n = \frac{1}{2\sin\phi}\left[\tan(n+1)\phi - \tan\phi\right]$$

$\therefore S = \frac{1}{2\sin 2\phi}\left[\tan(n+1)\phi - \tan\phi\right]$ ③

EXERCISE-7

① $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{2}{9} + \tan^{-1}\frac{4}{33} + \cdots + \tan^{-1}\frac{2^{n-1}}{1+2^{2n-1}}$

$t_n = \tan^{-1}\frac{2^{n-1}}{1+2^{2n-1}} = \tan^{-1}\frac{2^{n-1}(1)}{1+2^n\cdot 2^{n-1}}$

$t_n = \tan^{-1}\frac{2^{n-1}(2-1)}{1+2^n\cdot 2^{n-1}} = \tan^{-1}\frac{2\cdot 2^{n-1} - 2^{n-1}}{1+2^n\cdot 2^{n-1}}$

$= \tan^{-1}\frac{2^n - 2^{n-1}}{1+2^n\cdot 2^{n-1}}$

$t_n = \tan^{-1}2^n - \tan^{-1}2^{n-1}$

$t_1 = \tan^{-1}2 - \tan^{-1}2^0$

$t_2 = \tan^{-1}2^2 - \tan^{-1}2^1$

$t_3 = \tan^{-1}2^3 - \tan^{-1}2^2$

$\vdots$

$t_{n-1} = \tan^{-1}2^{n-1} - \tan^{-1}2^{n-2}$

$t_n = \tan^{-1}2^n - \tan^{-1}2^{n-1}$

Adding $S = \tan^{-1}2^n - \tan^{-1}1 = \tan^{-1}2^n - \pi/4$ Ans